

Scattering problem of scalar wave in wormhole geometry

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In this paper, we study the scattering problem of the scalar wave in the traversable Lorentzian wormhole geometry. The potentials and Schrödinger-like equations are found in cases of the static uncharged and the charged wormholes. The differential scattering cross sections are determined by the phase shift of the asymptotic wave function in low frequency limit. It is also found that the cross section for charged wormhole is smaller than that for uncharged case by the reduction of the throat size due to the charge effect.

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I. INTRODUCTION

The wormhole has the structure which is given by two asymptotically flat regions and a bridge connecting two regions [1]. For the Lorentzian wormhole to be traversable, it requires exotic matter which violates the known energy conditions. To find the reasonable models, there had been studying on the generalized models of the wormhole with other matters and/or in various geometries. Among the models, the matter or wave in the wormhole geometry and its effect such as radiation are very interesting to us. The scalar field could be considered in the wormhole geometry as the primary and auxiliary effects [2]. Recently the solution for the electrically charged case was also found [3]. Scalar wave solutions in the wormhole geometry [4,5] was in special wormhole model only the transmission and reflection coefficients were found. The electromagnetic wave in wormhole geometry is recently discussed [6] along the method of scalar field case. These wave equations in wormhole geometry draws attention to the research on radiation and wave. Also there was a suggestion that the wormhole would be one of the candidates of the gamma ray bursts [7].

For the gravitational radiation in any forms, the scattering problem to calculate the cross section in more generalized models of wormhole should be considered. Thus the study of scalar, electromagnetic, gravitational waves in wormhole geometry is necessary to the research on the gravitational radiation.

In this paper, we study the scalar wave in static uncharged and charged wormhole and find the differential scattering cross sections. We also compare the results each other in both cases to see the charge effect on scattering problem. Here we adopt the geometrical unit, *i.e.*, $G = c = \hbar = 1$.

II. STATIC UNCHARGED WORMHOLE

The spacetime metric for static uncharged wormhole is given as

$$ds^2 = -e^{2\Lambda(r)} dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (2.1)$$

where $\Lambda(r)$ is the lapse function and $b(r)$ is the wormhole shape function. They are assumed to be dependent on r only for static case.

The wave equation of the minimally coupled massless scalar field is given by

$$\square\Phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi) = 0. \quad (2.2)$$

In spherically symmetric space-time, the scalar field can be separated by variables,

$$\Phi_{lm} = Y_{lm}(\theta, \phi) \frac{u_l(r, t)}{r}, \quad (2.3)$$

where $Y_{lm}(\theta, \phi)$ is the spherical harmonics and l is the quantum angular momentum.

If $l = 0$ and the scalar field $\Phi(r)$ depends on r only, the wave equation simply becomes the following relation [8]:

$$e^\Lambda \sqrt{1 - \frac{b}{r}} r^2 \frac{\partial}{\partial r} \Phi = A = \text{const.} \quad (2.4)$$

In this relation, the back reaction of the scalar wave on the wormhole geometry is neglected. Thus the static scalar wave without propagation is easily found as the integral form of

$$\Phi = A \int e^{-\Lambda} r^{-2} \left(1 - \frac{b}{r}\right)^{-1/2} dr. \quad (2.5)$$

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The scalar wave solution was already given to us for the special case of wormhole in Ref. [3,8].

More generally, if the scalar field Φ depends on r and t , the wave equation after the separation of variables (θ, ϕ) becomes

$$-\ddot{u}_l + \frac{\partial^2 u_l}{\partial r_*^2} = V_l u_l, \quad (2.6)$$

where the potential is

$$\begin{aligned} V_l(r) &= \frac{L^2}{r^2} e^{2\Lambda} + \frac{1}{r} e^\Lambda \sqrt{1 - \frac{b}{r}} \frac{\partial}{\partial r} \left(e^\Lambda \sqrt{1 - \frac{b}{r}} \right) \\ &= e^{2\Lambda} \left(\frac{l(l+1)}{r^2} - \frac{b'r - b}{2r^3} + \frac{1}{r} \left(1 - \frac{b}{r} \right) \Lambda' \right) \end{aligned} \quad (2.7)$$

and the proper distance r_* has the following relation to r :

$$\frac{\partial}{\partial r_*} = e^\Lambda \sqrt{1 - \frac{b}{r}} \frac{\partial}{\partial r}. \quad (2.8)$$

Here, $L^2 = l(l+1)$ is the square of the angular momentum.

The properties of the potential are determined by the shape of it, if only the explicit forms of Λ and b are given. If the time dependence of the wave is harmonic as $u_l(r, t) = \hat{u}_l(r, \omega) e^{-i\omega t}$, the equation becomes

$$\left(\frac{d^2}{dr_*^2} + \omega^2 - V_l(r) \right) \hat{u}_l(r, \omega) = 0. \quad (2.9)$$

It is just the Schrödinger equation with energy ω^2 and potential $V_l(r)$. When $e^{2\Lambda}$ is finite, V_l approaches zero as $r \rightarrow \infty$, which means that the solution has the form of the plane wave $\hat{u}_l \sim e^{\pm i\omega r_*}$ asymptotically. The result shows that if a scalar wave passes through the wormhole the solution is changed from $e^{\pm i\omega r}$ into $e^{\pm i\omega r_*}$, which means that the potential affects the wave and experience the scattering. As $r \rightarrow b$ (near throat), the potential has the finite value of $V_l \simeq e^{2\Lambda(b)} \frac{l(l+1)+1/2}{b^2}$.

As the simplest example for this problem, we consider the special case ($\Lambda = 0, b = b_0^2/r$) as usual, the potential should be in terms of r or r_* as

$$V_l = \frac{l(l+1)}{r^2} + \frac{b_0^2}{r^4} \quad \text{or} \quad \frac{l(l+1)}{r_*^2 + b_0^2} + \frac{b_0^2}{(r_*^2 + b_0^2)^2}, \quad (2.10)$$

where the proper distance r_* is given by

$$r_* = \int \frac{1}{\sqrt{1 - b_0^2/r^2}} dr = \sqrt{r^2 - b_0^2}. \quad (2.11)$$

This is the hyperbolic relation between r_* and r which is plotted in Fig. 4. The potentials are depicted in Fig. 1. The potential has the maximum value as

$$V_l(r)|_{\max} = V_l(r_*)|_{\max} = V_l(b_0) = \frac{l(l+1)+1}{b_0^2}. \quad (2.12)$$

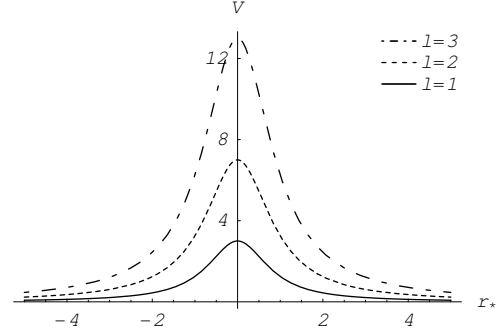


FIG. 1. Plot of the potentials of the specified wormhole $b = b_0^2/r$ for $l = 1, 2, 3$. Here we set $b_0 = 1$.

Now we can see the characteristics of the potential Eq. (2.10) in the specified limits. When $l = 0$, the potential and its asymptotic value are

$$V_l(r_*) = \frac{b_0^2}{(r_*^2 + b_0^2)^2} \xrightarrow{r_* \rightarrow \infty} \frac{b_0^2}{r_*^4}. \quad (2.13)$$

When $l \gg 1$, the potential and its asymptotic value are

$$V_l(r_*) \approx \frac{l^2}{r_*^2 + b_0^2} \xrightarrow{r_* \rightarrow \infty} \frac{l^2}{r_*^2}. \quad (2.14)$$

As $r_* \rightarrow \infty$ ($r_*^2 \gg b_0^2$) independently of the value of l , the potential has the asymptotic value as

$$V_l \simeq \frac{l(l+1)}{r_*^2} - \frac{[l(l+1)-1]b_0^2}{r_*^4}. \quad (2.15)$$

From the potential Eq. (2.10), the transmission coefficient can be calculated by WKB approximation as

$$|T|_{(l\omega)}^2 = \exp \left(-2 \int_{a_-}^{a_+} \left(\frac{l(l+1)}{r_*^2 + b_0^2} + \frac{b_0^2}{(r_*^2 + b_0^2)^2} - \omega^2 \right)^{1/2} dr_* \right), \quad (2.16)$$

where the upper and lower integration limits a_+ and a_- are

$$a_{\pm} = \pm \left(-b_0^2 + \frac{l(l+1)}{2\omega^2} + \frac{\sqrt{l^2(l+1)^2 + 4b_0^2\omega^2}}{2\omega^2} \right)^{1/2}. \quad (2.17)$$

The transmission coefficient means the probability that the scalar wave can pass through the throat of the wormhole, even though it does not have enough energy to overcome the potential. A few transmission coefficients are depicted in Fig. 2.

For large l , it becomes

$$|T|^2 \approx \left(\frac{\sqrt{l^2 + \omega^2 b_0^2} - l}{\sqrt{l^2 + \omega^2 b_0^2} + l} \right)^{2l}. \quad (2.18)$$

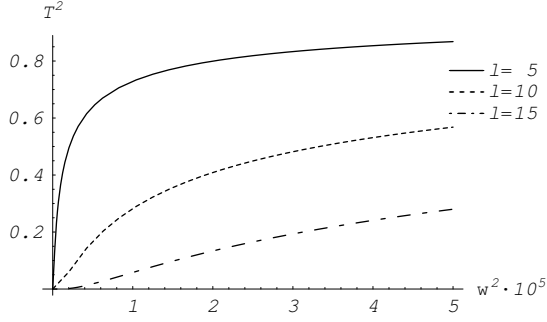


FIG. 2. The ω -dependency of $|T|^2_{(l\omega)}$ for the specified wormhole $b = b_0^2/r$ for $l = 5, 10, 15$. Here we set $b_0 = 1$.

When $l = 0$, it is exactly given as

$$|T|^2 = e^{-A(\omega)}, \quad (2.19)$$

where

$$A(\omega) = \frac{b_0^2 \pi}{\omega^2} \left(\frac{2}{b_0} - \frac{(\frac{\omega}{b_0})^{1/2} \sqrt{2\pi_3} F_2[\{-\frac{1}{4}, \frac{1}{4}, \frac{3}{4}\}, \{\frac{1}{2}, \frac{5}{4}\}; b_0^2 \omega^2]}{\Gamma(\frac{1}{4})\Gamma(\frac{5}{4})} + \frac{(\frac{\omega}{b_0})^{3/2} b_0^2 \sqrt{2\pi_3} F_2[\{\frac{1}{4}, \frac{3}{4}, \frac{5}{4}\}, \{\frac{3}{2}, \frac{7}{4}\}; b_0^2 \omega^2]}{\Gamma(-\frac{1}{4})\Gamma(\frac{7}{4})} \right). \quad (2.20)$$

Here ${}_pF_q(\{a_1, \dots, a_p\}, \{b_1, \dots, b_q\}; z)$ is the generalized hypergeometric function.

For more deep understanding on scattering problem, we should find the differential cross section. Since we discuss the low frequency case similar to the black hole case [9], we approach the problem with the different definition of Φ by using $R(r, t)$ instead of $u_l(r, t)/r$ unlike Eq. (2.3) as

$$\Phi = R(r, t)Y(\theta, \phi). \quad (2.21)$$

The equation, in terms of R , becomes

$$-e^{-2\Lambda} r^2 \ddot{R} + e^{-\Lambda} \sqrt{1 - \frac{b}{r}} \frac{\partial}{\partial r} \left(e^{\Lambda} \sqrt{1 - \frac{b}{r}} r^2 \frac{\partial}{\partial r} R \right) = l(l+1)R. \quad (2.22)$$

The equation can be rewritten as

$$\frac{d}{dr} \Delta \frac{dR}{dr} + \left(\frac{(r^2 \omega^2)^2}{\Delta} - l(l+1) \frac{e^{\Lambda}}{\sqrt{1 - b/r}} \right) R = 0, \quad (2.23)$$

where $\Delta = e^{\Lambda} \sqrt{1 - \frac{b}{r^2}}$. If we set $R = \Delta^{-1/2} u$, the first term of the equation Eq. (2.23) should be

$$\frac{d}{dr} \Delta \frac{dR}{dr} = \Delta^{1/2} \left\{ \frac{d^2 u}{dr^2} + \left(\frac{1}{4} \frac{1}{\Delta^2} \left(\frac{d\Delta}{dr} \right)^2 - \frac{1}{2} \frac{1}{\Delta} \frac{d^2 \Delta}{dr^2} \right) u \right\}. \quad (2.24)$$

Thus the equation Eq. (2.23) becomes as

$$\frac{d^2 u}{dr^2} + \left(\frac{(r^2 \omega^2)^2}{\Delta} - l(l+1) \frac{e^{\Lambda}}{\sqrt{1 - b/r}} + \frac{1}{4} \frac{1}{\Delta} \left(\frac{d\Delta}{dr} \right)^2 - \frac{1}{2} \frac{d^2 \Delta}{dr^2} \right) \frac{u}{\Delta} = 0. \quad (2.25)$$

For example, $\Lambda = 0$, $b = b_0^2/r$ like the potential problem, then Δ should be

$$\Delta = \sqrt{r^4 - b_0^2 r^2}. \quad (2.26)$$

The equation Eq. (2.25) is expanded in power of r as

$$\frac{d^2 u}{dr^2} + \left(\omega^2 + \frac{\omega^2 b_0^2 - l(l+1)}{r^2} + b_0^2 \frac{\omega^2 b_0^2 + \frac{1}{2} - l(l+1)}{r^4} + O(r^{-6}) \right) u \simeq 0. \quad (2.27)$$

As $r \rightarrow \infty$ neglecting the terms $O(r^{-4})$, the equation becomes

$$\frac{d^2 u}{dr^2} + \left(\omega^2 + \frac{\omega^2 b_0^2}{r^2} - \frac{l(l+1)}{r^2} \right) u \simeq 0. \quad (2.28)$$

When the frequency is enough low for being as $\omega^2 b_0^2 < l(l+1)$, the scattering is elastic and the solution to the equation is

$$u_{l\omega} = r j_{\lambda}(\omega r). \quad (2.29)$$

Here $\lambda(\lambda+1) = l(l+1) - \omega^2 b_0^2$ or $\lambda = -\frac{1}{2} + \sqrt{(l+\frac{1}{2})^2 - \omega^2 b_0^2}$ and $j_{\lambda}(r)$ is the spherical Bessel function. The turning point is

$$r_{\text{TP}} = \sqrt{\frac{l(l+1)}{\omega^2} - b_0^2}. \quad (2.30)$$

When $\omega^2 b_0^2 \geq l(l+1)$, there is a total absorption and no turning point. Since the solution Eq. (2.29) asymptotically becomes

$$u_{l\omega} \sim \frac{1}{\omega} \sin(\omega r - \frac{\pi}{2} \lambda), \quad (2.31)$$

the phase shift of the wave function by the potential is

$$\delta_l = -\frac{\pi}{2} \left(\sqrt{(l+\frac{1}{2})^2 - \omega^2 b_0^2} - \left(l + \frac{1}{2} \right) \right). \quad (2.32)$$

The phase shift δ_l is obviously positive, which means that the potential is attractive. The scattering amplitude in partial wave expansion is

$$f(\theta) = \frac{1}{\omega} \sum_{l=0}^{\infty} (2l+1) P_l(\cos \theta) e^{i\delta_l} \sin \delta_l. \quad (2.33)$$

If $\omega^2 b_0^2$ and δ_l are very small, the scattering amplitude is given by

$$\begin{aligned}
f(\theta) &\simeq \frac{1}{\omega} \sum_{l=0}^{\infty} (2l+1) P_l(\cos \theta) \delta_l \\
&\simeq \frac{\pi \omega b_0^2}{2} \sum_{l=0}^{\infty} P_l(\cos \theta) \\
&= \frac{\pi \omega b_0^2}{4} \frac{1}{\sin \frac{\theta}{2}}.
\end{aligned} \tag{2.34}$$

The relation

$$\delta_l \simeq \frac{\pi \omega^2 b_0^2}{2(2l+1)} \tag{2.35}$$

is used for the second line. The equation (2.35) is derived from Eq. (2.32) by using the fact that $\omega^2 b_0^2 \ll 1$. Therefore, the differential cross section is

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \frac{\pi^2 \omega^2 b_0^4}{16 \sin^2 \frac{\theta}{2}}. \tag{2.36}$$

In the case of very low frequency, $\frac{l}{b_0 \omega} \rightarrow \infty$ neglecting $b_0 \omega$ term, the equation Eq. (2.28) becomes

$$\frac{d^2 u}{dr^2} + \left(\omega^2 - \frac{l(l+1)}{r^2} \right) u \simeq 0, \tag{2.37}$$

which means no scattering, since any term is not appeared except energy ω^2 and centrifugal term. However, it is Coulomb scattering for black hole, since there is the term r^{-1} [9]. The solution to the equation for black hole is

$$u = j_l(\omega r). \tag{2.38}$$

If we remain the term r^{-4} with low frequency approximation in Eq. (2.27), then

$$\begin{aligned}
\frac{d^2 u}{dr^2} + \left(\omega^2 - \frac{l(l+1)}{r^2} + b_0^2 \frac{1/2 - l(l+1)}{r^4} \right) u \\
\simeq 0
\end{aligned} \tag{2.39}$$

The equation can be solved as the problem with an r^{-4} potential. In this case, the cross section is given in most text books on quantum mechanics [10]. In WKB approximation, the wave function has the asymptotic form of

$$u_l \sim D \sin \left(\int_{r_0}^r \sqrt{\omega^2 - U - \frac{(l+1/2)^2}{r^2}} dr + \frac{\pi}{4} \right), \tag{2.40}$$

where

$$U(r) = \frac{b_0^2 [l(l+1) - 1/2]}{r^4}. \tag{2.41}$$

The phase shift in this wave function should be

$$\begin{aligned}
\delta_l &= \lim_{R \rightarrow \infty} \left(\int_a^R \sqrt{\omega^2 - U - \frac{(l+1/2)^2}{r^2}} dr \right. \\
&\quad \left. - \int_{a_0}^R \sqrt{\omega^2 - \frac{(l+1/2)^2}{r^2}} dr \right) \\
&\simeq -\frac{1}{2} \int_{l/\omega}^{\infty} U(r) \left(\omega^2 - \frac{l^2}{r^2} \right)^{-1/2} = -\frac{b_0^2 \omega \pi}{4l},
\end{aligned} \tag{2.42}$$

where U is assumed to be very small in asymptotic region comparing to the two terms in the square root, even though l is defined as the large value. Here, the phase shift is negative and it is natural by the repulsive potential Eq. (2.41). With this phase shift, the differential cross section should be

$$\frac{d\sigma}{d\Omega} \simeq \frac{b_0^4 \pi^2}{16 \sin^2 \frac{\theta}{2}}. \tag{2.43}$$

This is restrictedly calculated in the limit of large angular momentum and small U . As we see, it is independent of ω and the θ -dependence is the same as that of the r^{-2} potential case, Eq. (2.36).

III. CHARGED WORMHOLE

The electrically charged wormhole is given by [3]

$$\begin{aligned}
ds^2 &= - \left(1 + \frac{Q^2}{r^2} \right) dt^2 + \left(1 - \frac{b(r)}{r} + \frac{Q^2}{r^2} \right)^{-1} dr^2 \\
&\quad + r^2 (d\theta^2 + \sin^2 \theta d\phi^2).
\end{aligned} \tag{3.1}$$

When $Q = 0$, this becomes the Morris-Thorne type wormhole spacetime [1] and when $b = 0$, this becomes the Reissner-Nordström black hole with zero mass. If we replace $b - \frac{Q^2}{r}$ by b_{eff} as

$$b \rightarrow b_{\text{eff}} = b - \frac{Q^2}{r}, \tag{3.2}$$

it self-consistently satisfies the field equations for the wormhole without Q . When $b = b_0^{\frac{2\beta}{2\beta+1}} r^{\frac{1}{2\beta+1}}$, the radius which limits the range of the wormhole becomes

$$r_0 = Q^{\frac{2\beta+1}{\beta}} b_0^{\frac{\beta}{2\beta+1}}. \tag{3.3}$$

For the special case of $\beta = -1$ or $b = b_0^2/r$, $Q^2 < b_0^2$ is the condition that is required for maintaining the wormhole under the addition of the charge.

The metric for the charged case with scalar field is also given by [3]

$$\begin{aligned}
ds^2 &= -dt^2 + \left(1 - \frac{b(r)}{r} + \frac{\alpha}{r^2} \right)^{-1} dr^2 \\
&\quad + r^2 (d\theta^2 + \sin^2 \theta d\phi^2).
\end{aligned} \tag{3.4}$$

The substitution $b \rightarrow b_{\text{eff}} = b - \frac{\alpha}{r}$ does not change the equation like the electrically charged case and $\alpha < b_0^2$ is also the condition for being safe under the addition of scalar charge.

To see the scalar wave in the charged wormhole, we first analyze the properties of the potential. In this case, only the following substitutions will be enough for the study:

$$e^{2\Lambda} = \left(1 + \frac{Q^2}{r^2}\right) \quad \text{and} \quad b \rightarrow b_{\text{eff}} = b - \frac{Q^2}{r} \quad (3.5)$$

For the special case of $b = \frac{b_0^2}{r}$, the proper length will be very complicated form as

$$r_* = \int \frac{1}{e^\Lambda \sqrt{1 - b_{\text{eff}}/r}} = \int \frac{r^2 dr}{\sqrt{(r^2 + Q^2)(r^2 + Q^2 - b_0^2)}} \\ = \sqrt{Q^2 - b_0^2} E\left(\sin^{-1} \frac{\sqrt{b_0^2 - r^2 - Q^2}}{b_0} \middle| -\frac{b_0^2}{b_0^2 - Q^2}\right), \quad (3.6)$$

where $E(\alpha|\beta)$ is the Elliptic integral of the second kind defined by

$$E(\alpha|\beta) = \int_0^\alpha \sqrt{1 - \beta \sin^2 \theta} d\theta. \quad (3.7)$$

With the substitution Eq. (3.5), the potential will be

$$V_l(r) = \left(1 + \frac{Q^2}{r^2}\right) \frac{l(l+1)}{r^2} + \left(1 + \frac{2Q^2}{r^2}\right) \frac{1}{r^4} (b_0^2 - Q^2) \\ - \frac{Q^2}{r^4} > 0. \quad (3.8)$$

If we examine the proper length r_* and potential V_l for the limit of infinite distance ($r \rightarrow \infty$) and near the throat of the wormhole ($r^2 \rightarrow b_0^2 - Q^2$), we can figure out the rough form of the potential. Since

$$\lim_{r \rightarrow \infty} r_* = \pm\infty \quad \text{and} \quad \lim_{r^2 \rightarrow b_0^2 - Q^2} r_* = 0, \quad (3.9)$$

the proper length has the similar form to the uncharged case. At the asymptotic region, the potential approach

$$\lim_{r \rightarrow \infty} V(r), V(r_*) = 0. \quad (3.10)$$

Since

$$\frac{dV}{dr_*} = \frac{dV}{dr} \frac{dr}{dr_*} = \frac{dV}{dr} \sqrt{\left(1 + \frac{Q^2}{r^2}\right) \left(1 - \frac{b_0^2}{r^2} + \frac{Q^2}{r^2}\right)} \quad (3.11)$$

and $\frac{dV}{dr_*} = 0$ when $\frac{dV}{dr} = 0$, the potential at the throat has a finite common maximum value as

$$\lim_{r \rightarrow b_0^2 - Q^2} V(r), V(r_*) = V(r)|_{\text{max}}, V(r_*)|_{\text{max}}. \quad (3.12)$$

The potential form is compared with the case of $Q^2 = 0$, so that

$$V_{\text{max}}|_{Q^2 \neq 0} = \frac{l(l+1)}{(b_0^2 - Q^2)} \left(1 + \frac{Q^2}{(b_0^2 - Q^2)}\right) + \frac{b_0^2}{(b_0^2 - Q^2)^2} \\ > \frac{l(l+1)}{b_0^2} + \frac{1}{b_0^2} = V_{\text{max}}|_{Q^2=0}. \quad (3.13)$$

As we see in Eq. (3.13), the charge effect contract the throat size of wormhole and it causes the maximum of the potential to be increased. Since the term in the bracket of the following equation is positive for $l \geq 1$ in general, the potentials of both cases are given as

$$V|_{Q^2 \neq 0} = \frac{l(l+1)}{r^2} + \frac{b_0^2}{r^4} + \frac{Q^2}{r^4} \left(l(l+1) - 2 + \frac{2}{r^2}(b_0^2 - Q^2)\right) \\ > V|_{Q^2=0}. \quad (3.14)$$

With these results, the potentials are plotted in Fig. 3 so that the potential for $Q^2 \neq 0$ is above the potential for $Q^2 = 0$. For the relation between r_* and r ,

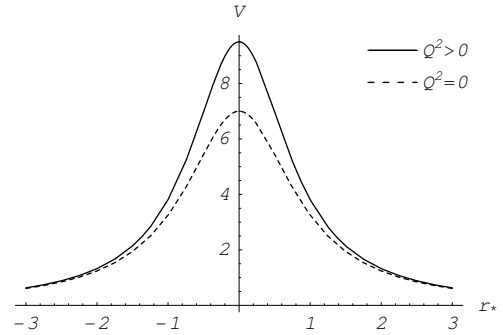


FIG. 3. Potentials for charged ($Q^2 \neq 0$) and uncharged ($Q^2 = 0$) wormholes. Here we set $b_0 = 1$, $Q^2 = 0.5$, and $l = 2$.

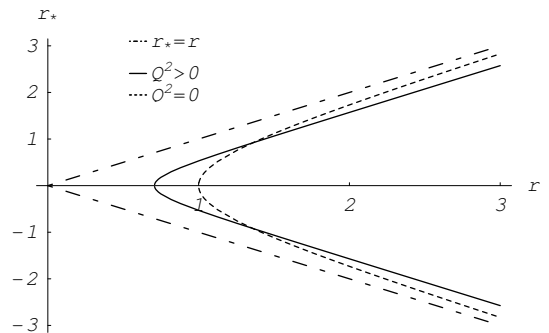


FIG. 4. The relation between r and r_* for charged ($Q^2 \neq 0$) and uncharged ($Q^2 = 0$) wormholes. Here we set $b_0 = 1$ and $Q^2 = 0.5$.

$$\begin{aligned} \frac{dr_*}{dr}|_{Q^2 \neq 0} &= \left(1 + \frac{Q^2}{r^2}\right)^{-1} \left(1 - \frac{b_0^2}{r^2} + \frac{Q^2}{r^2}\right)^{-1/2} \\ &< \left(1 - \frac{b_0^2}{r^2}\right)^{-1/2} = \frac{dr_*}{dr}|_{Q^2=0} \quad (3.15) \end{aligned}$$

is plotted in Fig. 4. It is based on the relations of r_* and r , *i.e.*, Eqs. (2.11) and (3.6) for charged and uncharged wormhole. At asymptotic region, $\lim_{r \rightarrow \infty} r_* = \pm r$ in both cases.

Since $V_{Q^2 \neq 0} > V_{Q^2=0}$ in the form of the potentials, the transmission coefficient is $|T|_{Q^2 \neq 0}^2 < |T|_{Q^2=0}^2$, which seems that the scalar wave can transmit harder in charged wormhole than neutral uncharged wormhole.

As $r \rightarrow \infty$ neglecting the terms $O(r^{-4})$, the equation becomes as

$$\frac{d^2 u}{dr^2} + \left(\omega^2 + \frac{\omega^2(b_0^2 - 3Q^2)}{r^2} - \frac{l(l+1)}{r^2} \right) u \simeq 0. \quad (3.16)$$

Since the b_0^2 term is simply replaced by $b_0^2 - 3Q^2$, the scattering cross section should be

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \frac{\pi^2 \omega^2 (b_0^2 - 3Q^2)^2}{16 \sin^2 \frac{\theta}{2}} < \frac{d\sigma}{d\Omega}|_{Q^2=0}. \quad (3.17)$$

In case of wormhole with scalar field, because only $\Lambda = 0$ is considered as we see in Eq. (3.4), the term $b_0^2 - 3Q^2$ for electrically charged case is just $b_0^2 - \alpha$ for scalar field case, so that the cross section should be

$$\frac{d\sigma}{d\Omega} = \frac{\pi^2 \omega^2 (b_0^2 - \alpha)^2}{16 \sin^2 \frac{\theta}{2}} < \frac{d\sigma}{d\Omega}|_{\alpha=0}. \quad (3.18)$$

Of course, the interaction of the wave with the scalar field is neglected here. In both cases, cross sections are reduced by the charge effect.

IV. DISCUSSION

Here we studied the scattering problem for the scalar wave in static uncharged and charged wormholes. The interactions of the scalar wave with the charge are neglected. If we consider them, the effect would make sense. The scattering cross sections are found and the charge effects are examined in this scattering problem. As further research, we will try other waves for various wormholes, such as the rotating wormhole and cosmological model with wormhole.

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